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Under this project we developed and demonstrated new techniques for identification and adaptive control. For linear identification, we developed quadratically constrained least squares (QCLS) identification, which extends classical least squares identification and is more resistant to the effects of system noise. For nonlinear identification, we developed a sequential method for combined Hammerstein-nonlinear feedback models. This method uses a piecewise linear approximation to the system nonlinearity and is based on computationally efficient numerical procedures. For adaptive control, we developed controllers for adaptive disturbance rejection and adaptive stabilization. These methods were implemented on laboratory experiments involving active noise control and rotating imbalance stabilization.

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Experimental and Theoretical Techniques for Nonlinear Identification and Adaptive Cancellation

Final Report

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1 Project Overview

Applications of control-related technology are increasing exponentially in both the military and civilian sectors. These applications range from "device" scale (for example, precision manufacturing and hard drive control) to "system" scale (for example, UAV technology and the Airborne Laser program). These problems are important, challenging, and have near-term relevance. In these and other applications, control technology has an excellent opportunity to make a major impact on system performance.

This project focused on research topics that we believe can contribute substantially to this technology, namely, *nonlinear identification* and *adaptive control*. Our choice of problems and techniques is based on the belief that fundamental, system-level (as distinct from technology-specific) research in these areas can impact a broad range of technology. While the proposed research is not confined to any one application area, our primary focus is on problems of vibration suppression (both acoustic and structural) as well as gyroscopic motion (spacecraft attitude control).

While mathematical rigor is essential to high quality control-related research, our program incorporates a strong element of experimentation. Experience has shown us that control experiments can provide the opportunity to test theoretical advances and determine their performance on real hardware. While the correctness of mathematical results is not subject to debate, the performance of identification and control algorithms on real data and hardware can be affected by innumerable phenomena that no mathematical theory can take into account. This statement does detract from the importance of mathematical research, which, in fact, provides the foundation for the major advances brought forth by the theoretical control community. Rather, we emphasize that control experiments can be crucial for identifying the underlying assumptions and physical features that an effective control theory must strive to address.

The control experiments we developed under this and related AFOSR projects [1, 4, 10, 13, 14, 18, 34, 55, 56, 73, 75, 77, 101, 113, 147, 151, 172, 174, 176] have played a critical role in shaping our thinking about research directions. These experiments were chosen to motivate our theoretical control research, to help set priorities, and to serve as an arbiter for determining which control algorithms were promising. Most importantly, experiments have helped us to determine *why* an approach did not work as we expected. This knowledge was sometimes obtained quickly, while at other times the source of the difficulty became clear only after a great deal of effort. We have found this dimension of control research to be extremely beneficial. Control experiments have also suggested new control ideas and techniques that would not have been suggested by theory and simulation alone. Examples include the virtual absorber resetting technique in [37] as well as the ARMARKOV adaptive disturbance rejection controller in [172, 174]. At the very least, control experiments force us to address implementation issues and hardware constraints, which may be overlooked in a purely theoretical setting. My article [13] stressed the positive role of control experiments within the context of theoretical control research.

Our control experiments have been designed to focus on system-level (rather than technology-specific) issues. By focusing on issues such as uncertainty, nonlinearity, dimensionality, and hardware constraints (for example, actuator saturation and sensor noise), our goal is to develop fundamentally new ideas and techniques that will have broad impact on control technology. At the same time, an additional benefit of control experiments is the opportunity to demonstrate to control practitioners how a newly developed technique performs on real hardware. This is the vision and philosophy underlying our combination of theoretical and experimental research. A detailed discussion of this viewpoint is given in [14].

The focus of this project on nonlinear identification and adaptive control was motivated by our hardware experience, which has shaped our view of which research directions can potentially make as significant impact on control applications. In particular, we have found that having a high-fidelity model of a dynamic system is not always critical for successful control system implementation. In fact, in certain cases, an adaptive control algorithm can significantly alleviate the need for both analytical and empirical modeling for controller tuning. This savings in modeling effort has motivated our interest in adaptive control. In addition, empirical modeling, that is, system identification, is of considerable significance in its own right for prediction, validation, and failure detection apart from control-system design and analysis.

2 Linear Identification

Linear identification theory has been extensively developed, and these techniques play an important role in applications. Methods for using data to construct models are based on least squares techniques, nonlinear programming (predictive error and maximum likelihood methods), frequency domain methods, set membership methods, subspace and realization techniques, and frequency domain methods [43, 47, 59, 70, 40, 87, 107, 109, 110, 121, 122, 123, 71, 72, 161, 166, 138, 159]. Relevant themes and issues in this research include recursive versus batch methods, time-domain versus frequency-domain techniques, bounded versus stochastic noise models, bias and consistency for stochastic modeling, over- and undermodeling, model stability, computational tractability, model validation, and robust-control-oriented identification.

Our research in linear identification has focused on extensions of least squares identification. While least squares identification is convenient for implementation, it lacks guarantees of consistency (that is, probability-1 convergence of parameter estimates) obtainable in principle from predictive error and maximum likelihood methods. Unfortunately, these latter methods depend on nonconvex optimization and thus require a search for the global minimizer. On the other hand, least squares identification is able to guarantee consistency for only a restrictive class of noise models, namely, systems with equation error. Consistency is also guaranteed for the numerator coefficients of an FIR (finite impulse response) model when this model structure is used within least squares ([108], p. 205). For an FIR model, the numerator coefficients are precisely the Markov parameters of the system. Interestingly, this consistency result is valid even if the identified system is actually an IIR (infinite impulse response) system.

In this project we developed two extensions of least squares identification for the purpose of extending guarantees of consistency. The first extension involves the use of an overparameterized (nonminimal) but sparse ARMAX model structure, called *ARMARKOV* models. The ARMARKOV model structure may be IIR, but includes FIR models (which are also sparse) as an extreme case. The interesting feature of these models is the fact that a subset of the numerator coefficients consists of Markov parameters of the system, and, like FIR model structures, these parameters can be consistently estimated by least squares identification. A gradient-based technique with explicit step size was developed in [6, 7] using ARMARKOV models. As an extension of the known result for least squares identification using FIR models, we showed in [167] that least squares estimates of the Markov parameters obtained using ARMARKOV models are consistent.

Next, we developed a fundamental extension of least squares identification called *quadratically constrained least squares (QCLS)*. This technique, developed in [170], removes a seemingly innocuous constraint assumed in classical least squares to obtain a fundamental extension of the standard theory. Specifically, since the numerator and denominator polynomials can be simultaneously scaled without changing the transfer function of the system, the leading denominator coefficient is traditionally normalized to unity. Of course, other normalizations are possible, for example, the combined vector of numerator and denominator coefficients can be normalized to unity with respect to an arbitrary norm. Intuition suggests that all such normalizations are equivalent in which case the classical normalization would entail no loss of generality. It was observed in [51], however, that alternative normalizations can eliminate frequency domain bias in the case of under- or over-modeling. An alternative normalization used in [106] was observed to entail bias in pole locations.

In QCLS theory we systematically investigated the implications of alternative normalizations for parameter consistency. The approach of [170] involves the minimization of a quadratic subject to a quadratic form constraint. Suitable choice of the side constraint yields standard least squares identification, while other choices enforce alternative normalizations of the transfer function coefficients. Computationally, the resulting numerical procedure is a tractable generalized eigenvalue problem. This problem alleviates the need for matrix inversion (or generalized inversion) required by standard least squares identification.

The main result of [170] guarantees parameter consistency under significantly more general conditions than standard least squares. Specifically, assuming that the noise autocorrelation function is known to within a constant multiple (but is otherwise arbitrary), QCLS yields consistent estimates for a general Box-Jenkins model. Figure 1 shows the improvement of QCLS parameter estimates over standard least squares. Note that the assumption of known noise statistics within a Box-Jenkins model does not assume knowledge of the

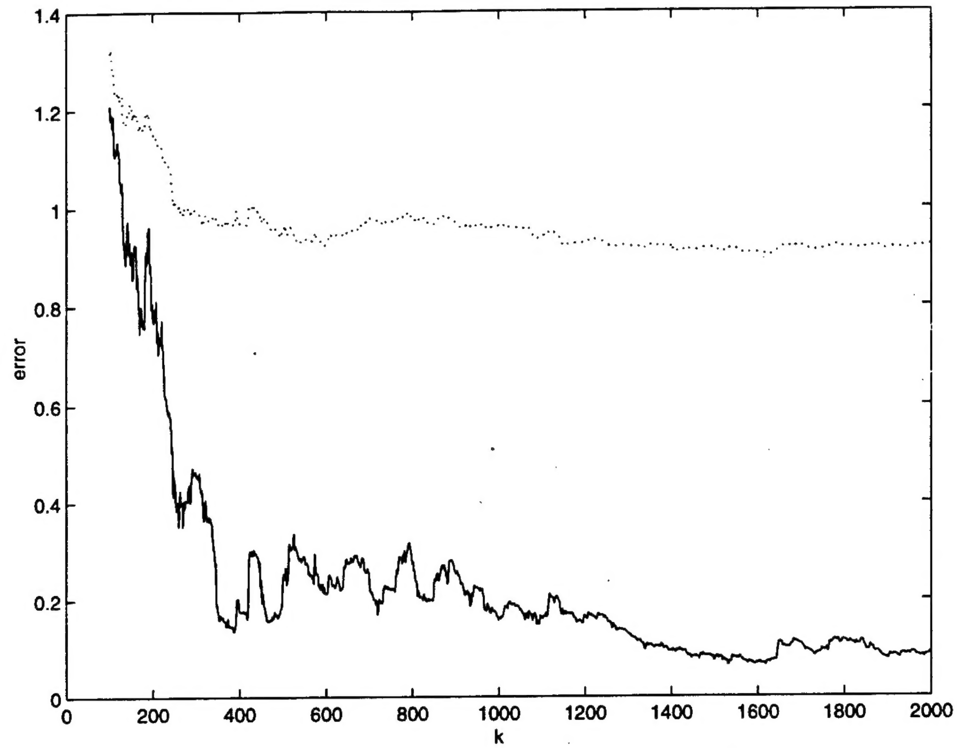

Figure 1: Quadratically constrained least squares identification (QCLS). For this example, which involves high noise levels, QCLS yields better estimates of the transfer function coefficients than standard least squares identification. The QCLS estimator is consistent for known noise statistics, which is not generally the case for standard least squares.

plant denominator dynamics as is the case with the more restrictive equation error model.

The consistency guarantee of QCLS provides an alternative to instrumental variables methods [160, 163]. These methods depend on the construction of a left inverse of the regressor matrix. This left inverse, which has a special form depending on the "instrument," is chosen to be uncorrelated with the measurement noise. Standard least squares theory chooses the pseudo inverse in an attempt to minimize the Frobenius norm of the left inverse. However, when statistical information is available concerning the measurement noise, it is often possible to construct a left inverse that is uncorrelated with the least squares residual (that is, bias). Instrumental variables provides methods for constructing uncorrelated left inverses. Unfortunately, however, there are no techniques for choosing the instrument with prior guarantees of success, and the available methods are largely ad hoc. The consistency of QCLS provides an alternative procedure that circumvents the need for constructing uncorrelated instruments.

In practice, the noise statistics are rarely known, and this provides an impediment to both QCLS and instrumental variables. Consequently, in [170] we used the QCLS algorithm as the basis for a heuristic, iterative technique involving successive refinement, and the resulting error is determined by an eigenvector perturbation problem. Proof of convergence of this algorithm is the subject of ongoing research. Relevant works include [48, 49, 50]. Additional open problems include model order determination. Relevant techniques include the lattice filter model structure for estimating and recursively modifying the system order [84]. Extensions to MIMO systems are being considered in ongoing research within the context of subspace methods [121, 122, 123].

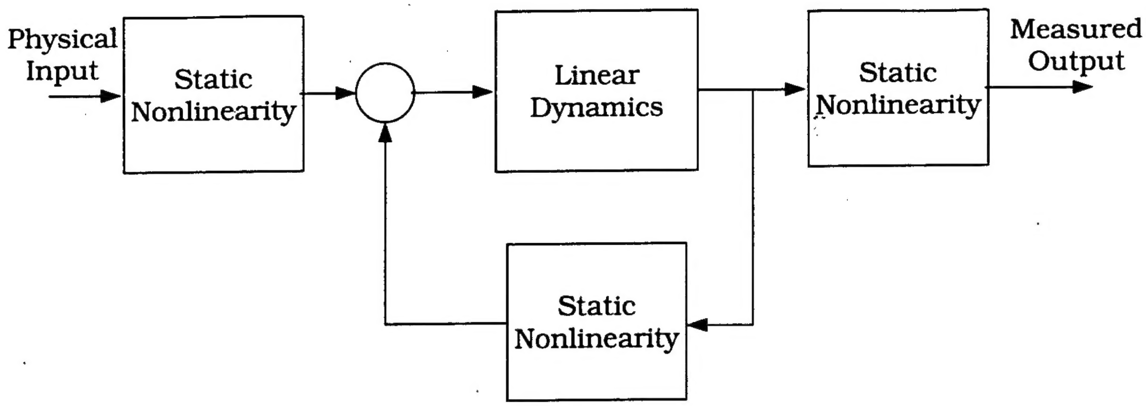

Figure 2: Block-structured models for nonlinear identification. These structured models involve the cascade and feedback interconnection of linear dynamic blocks and static nonlinear blocks.

3 Nonlinear Identification

In practice, all real systems are nonlinear, and the extent to which the nonlinearities require detailed modeling is application dependent. It is clear, however, that there are many applications in which the presence of hidden nonlinearities can render linear identification methods inappropriate. In our experience, we have found that relatively mild nonlinearities can impact the ability to perform linear identification more severely than significant noise levels. Consequently, we consider nonlinear identification to be a research area of high priority.

The literature on nonlinear identification is fairly extensive, but limited compared to linear identification and when viewed against its practical ramifications and potential scope. Among the many approaches that have been proposed for nonlinear identification we can mention Volterra methods, neural net modeling, and block-structured models. Nonlinear identification based on Volterra models has been studied extensively in the early literature, and a summary is given in Chapter 11 of [145]. An alternative approach of considerable interest is the use of neural networks. These systems provide efficient methods for multivariable function approximation [46, 63, 78, 136] and have been applied extensively to nonlinear system identification [41, 111, 131]. Both Volterra and neural net methods can be viewed as black box models, that is, models that assume no prior knowledge concerning the internal model structure [88, 157].

In many engineering applications it is desirable to construct models that give insight into the constituent components of a nonlinear system. For example, a mechanical system is expected to have a dissipation term that depends on position and velocity, while an electromechanical actuator has a velocity-dependent back EMF characteristic. Additional examples include actuators with saturation, sensors with nonlinear distortion, and nonlinear modal coupling in a flexible structure. Because of their engineering interpretation, it is desirable to identify these constituent components as separate entities. The drawback of black box models is their inability to make this component distinction. Such techniques also lack the ability to exploit information about the plant structure, which renders them computationally inefficient when such information is available. On the other hand, we recognize that such models are desirable when model structure is not of interest, is unknown, or is highly complex.

Consequently, our interest in component models, has led us to consider *block-structured models* involving the cascade and feedback interconnection of two types of blocks, namely, linear dynamic blocks and static nonlinear blocks. Some representative structures are shown in Figure 2. Block-structured models have been widely studied, and various identification techniques have been proposed [11, 28, 29, 30, 33, 39, 52, 68, 69, 79, 80, 94, 96, 130, 133, 134, 135, 137, 146, 155, 162, 171, 179].

In this project, we developed an approach to nonlinear identification for Hammerstein/nonlinear feedback models that extends a technique developed [11]. Bai developed an identification algorithm for Hammerstein

systems, that is, block-structured models with a static nonlinear block preceding a linear dynamic block. The appealing aspect of this algorithm is the fact that the need for nonlinear optimization is replaced by a 2-stage procedure involving computationally tractable procedures. Therefore, unlike nonlinear programming algorithms in general, this algorithm does not require computationally demanding line search routines.

In [168, 169], we provided an interpretation of the 2-stage algorithm, and we extended it to nonlinear feedback. Specifically, we derived a bound for the identification error and showed that the components of the bound can be minimized sequentially, that is, the outcome of the first optimization step serves as a parameter vector for the subsequent optimization step. This procedure essentially serves to disentangle the parameters of the static nonlinear block from the transfer function coefficients of the dynamic linear block.

Of special interest is the fact that these intermediate optimization problems are tractable, consisting of a standard least-squares problem followed by a low-rank approximation in a unitarily invariant norm solvable by a singular value decomposition. The relevant theory and computational procedure have been developed in [168] for the combined Hammerstein/nonlinear feedback model.

A crucial observation concerning the algorithm developed in [11] is the fact that, for some parameterizations of the static nonlinearity, the regressor for the standard least squares step is rank deficient, that is, equivalent to a least squares problem without persistent excitation. This obstacle was overcome in [168] by employing a piecewise linear function approximation parameterized in terms of a function value and slopes. For reasons that remain to be investigated, this parameterization of a piecewise linear approximation leads to a well-posed least squares procedure, whereas other parameterizations do not. For the nonlinear feedback identification problem, the parameterization must satisfy additional conditions, which hold for the point-slope parameterization.

For illustration, Figure 3 shows the results of a nonlinear identification procedure applied to a system with a hidden feedback nonlinearity. For this example we constructed a piecewise linear approximation of the static nonlinear block using the point-slope parameterization discussed above.

We have also considered identification of Wiener systems, which are dual to Hammerstein systems. Wiener systems involve a static nonlinearity immediately preceding the output [29, 39, 69, 79, 133, 181]. We note that the "Wiener" model of [11] is not a true Wiener system, however, and thus this extension remains open.

While there is considerable literature on the identification of Wiener systems, virtually all of these methods are based on the assumption that the output nonlinearity is one-to-one. However, it is often the case that the sensor nonlinearity is not one-to-one. For example, sensor saturation with the usual "flat" characteristic is not one-to-one, as is the case with sensors that provide sign information only [92, 93]. In the non-one-to-one case, complete identification of the linear dynamic portion of the model is not possible.

For the Wiener problem with a non-one-to-one output nonlinearity we considered the case of a known output nonlinearity but with unknown linear dynamics. The two-stage algorithm developed in [99]has been demonstrated for simulated examples with sign, saturation, and quantization output nonlinearities. Current research is focusing on the more difficult case of unknown non-one-to-one output nonlinearity and unknown linear dynamics.

Ongoing work is also focusing on the extension to MIMO multivariable identification. A crucial problem is the choice of basis function. One popular approach is to work with radial basis functions [42, 111, 136]. The ability to perform multivariable nonlinear identification suggests the use of multivariable static blocks with current and past outputs as inputs. This practice blurs the distinction between static and dynamic blocks, and it allows one to capture unstructured (mixed dynamic and static) uncertainty by means of a black box model. Unfortunately, the computational complexity grows quickly for high-dimensional inputs and outputs, as it does with neural net model structures.

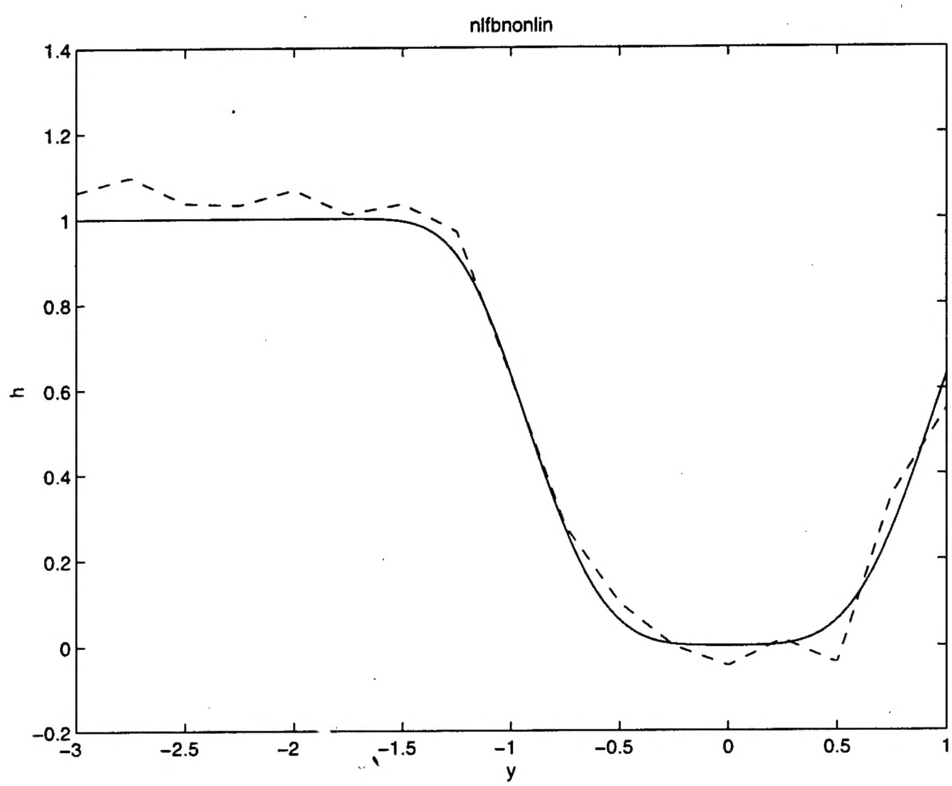

Figure 3: Piecewise linear least squares identification of a nonlinear feedback system. This example illustrates the use of nonlinear identification to identify a hidden static nonlinearity interconnected by feedback with a linear system. A piecewise-linear approximation was obtained for the static nonlinearity in the presence of sensor noise. The identification algorithm simultaneously identifies both the linear and nonlinear system components.

4 Adaptive Stabilization and Command Following

Our research in adaptive control includes problems in adaptive stabilization and command following as well as adaptive disturbance rejection. The stabilization and command following problem is discussed in the section, while the disturbance rejection problem is discussed separately in the following section.

By adaptive stabilization and adaptive command following we include model reference adaptive control but we exclude problems with unknown exogenous disturbances. The most basic problem (although nontrivial both theoretically or practically) is the problem of adaptive stabilization. In this problem the goal is to devise a controller that can stabilize the plant (that is, have all states approach the zero trajectory) under extremely limited modeling information. Adaptive controllers are able to achieve this objective through continual gain adjustment.

The literature on adaptive stabilization and adaptive command following is extensive; classical texts include [9, 83, 90, 95, 129, 154]. Most of this literature involves certain key assumptions, namely, that the plant has known relative degree, the plant is minimum phase, and the sign of the high frequency gain of the plant is known (more precisely, the leading coefficient of the numerator has known sign). Notice that all of these assumptions concern the phase, rather than the gain of the plant. Various weakenings of these assumptions have been achieved in the universal stabilization literature, albeit at the expense of considerably greater complexity [60, 81, 82, 118, 119, 120, 125, 126, 128, 132]. We have found these controllers difficult to implement experimentally due to large state transients.

Our research on adaptive stabilization was motivated by several control experiments that we developed. Using standard Lyapunov techniques we developed a full-state feedback controller for command following in [76], and we applied this controller to an electromagnetically actuated testbed and a servopneumatic project [75, 77]. The research on pneumatic control project was sponsored by HR Textron, Inc., while the adaptive controller has been implemented by engineers at Honeywell, Phoenix, AZ, for controller tuning. Computational and hardware experiments showed that the controller is highly effective against unmodeled stiffness, damping, and input and output nonlinearities.

The effectiveness of this adaptive controller on nonlinear systems led to the results of [143] which proved that the controller is guaranteed to stabilize second-order nonlinear systems in position-velocity coordinates with position-dependent but otherwise nonparametric (unstructured) damping and stiffness. Figures 4 and 5 show the position-velocity phase plane trajectories for the Van der Pol and Duffing oscillators. In both cases the adaptive controller was given no information concerning the damping and stiffness nonlinearities.

The next step in our development was to reduce the requirement for full-state feedback. To this end, we considered an output feedback stabilization problem for a second-order plant with relative degree zero or one. If the plant has relative degree 2 or if it is minimum phase and has relative degree 1, then it is not high-gain stabilizable as in the case of full state feedback. Although adaptive controllers for this problem under minimum phase assumptions are outlined in [129, 154], the construction of an explicit controller does not appear in the literature. To fill this gap, we developed an extended Lyapunov technique in which the Lyapunov derivative is asymptotically nonnegative [150]. This method guarantees attractivity, but not Lyapunov stability. As can be seen from Figure 6, this controller has some unexpected effectiveness for nonlinear plants.

Our research in adaptive stabilization and command following encompasses problems involving gyroscopic dynamics. In earlier work we developed controllers for the spinning top [112, 177, 178] and rotating shaft [114]. To support this work we developed an adaptive stabilization experiment involving a rotating shaft stabilized by a magnetic bearing actuator [113]. More recently, in [3, 5] we developed an adaptive command following controller for a spacecraft with unknown mass distribution (including unknown moments of inertia and center of mass). In [1, 4, 2] we developed similar controllers for a dual-gimbal control-moment gyro and a rigid rotor. To motivate and test our ideas, we constructed an actively controlled control-moment gyro (CMG) (see Figure 7). Attached to a spacecraft and with a spinning rotor, a CMG uses gyroscopic effects to provide stiffness to the spacecraft and, by applying torques to the gimbals, can be used to perform slewing maneuvers. In practice, however, an unbalanced CMG can cause disturbance forces known as "wheel noise."

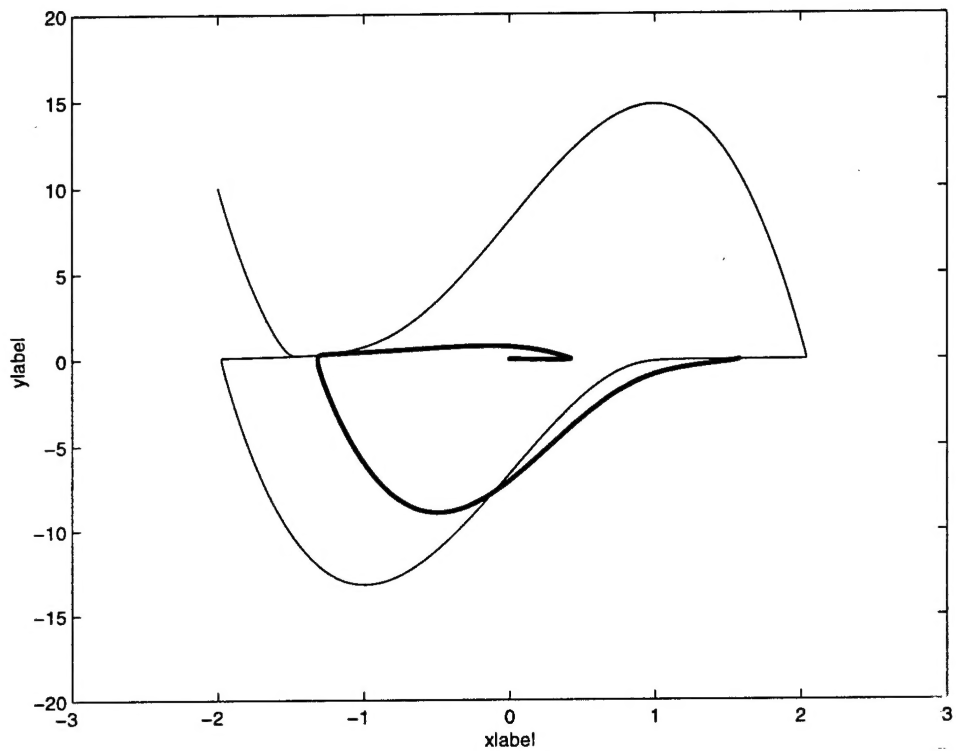

Figure 4: Full-state-feedback adaptive stabilization of van der Pol's oscillator. This is a plot of the phase plane trajectories of van der Pol's oscillator under full-state-feedback direct adaptive control. After the uncontrolled system reaches its limit cycle the adaptation is started and the origin is stabilized. The adaptive controller had no knowledge of the damping nonlinearity. Attractivity was guaranteed by a Lyapunov construction.

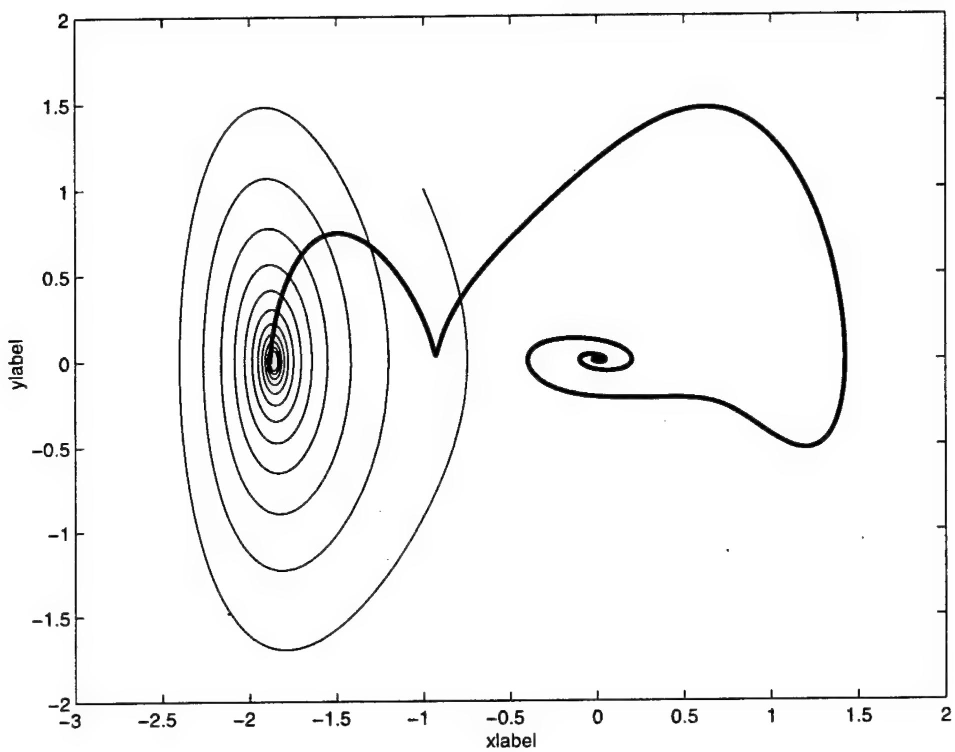

Figure 5: Full-state-feedback adaptive stabilization of the origin for Duffing's oscillator with multiple open-loop equilibria. This is a plot of the phase plane trajectories of the Duffing oscillator under full-state-feedback direct adaptive control. After the system approaches the equilibrium at (-1,0), the adaptation is started and the origin is stabilized. The adaptive controller had no knowledge of the functional form of the stiffness nonlinearity.

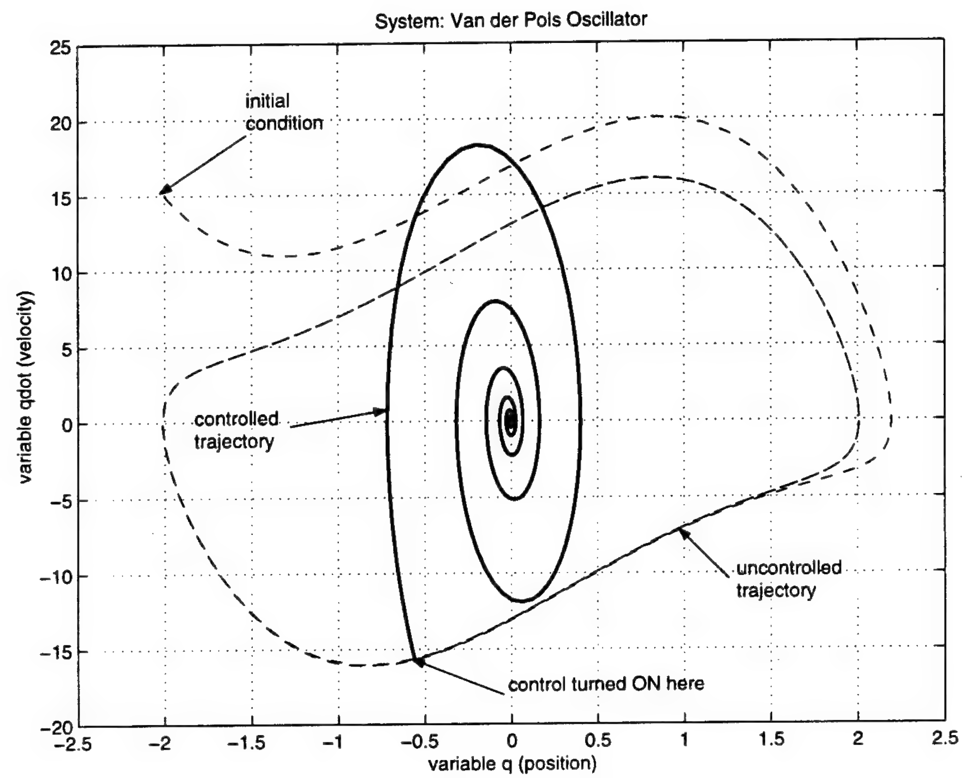

Figure 6: Output-feedback adaptive stabilization of van der Pol's oscillator. In this example, van der Pol's oscillator is adaptively stabilized using position measurement only. The 8th-order controller was developed for linear systems having relative degree 2.

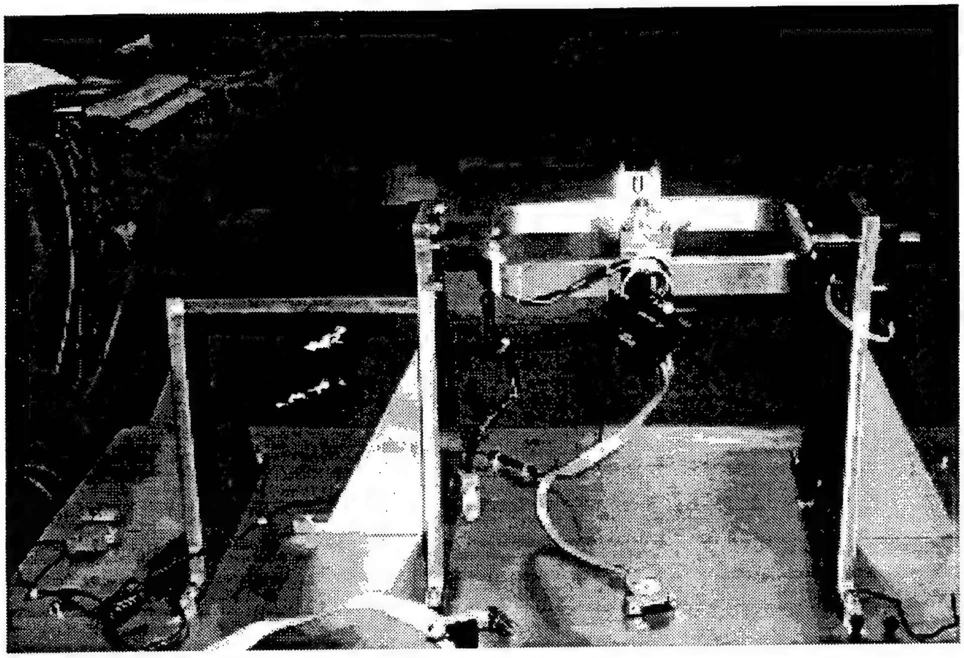

Figure 7: Dual gimbal control moment gyro (CMG) experiment. This testbed was developed to test adaptive stabilization control laws for gyroscopic systems with unknown mass imbalance.

The adaptive controller was able to follow wheel position commands without modeling of the wheel and gimbal mass distribution (see Figure 8).

Our interest in control of rotational systems motivated a study of the impossibility of global stabilization on compact manifolds. A detailed analysis of this problem was presented in [25].

For discrete-time systems we proved convergence of a full-state-feedback adaptive stabilization algorithm [175]. The key feature of this controller is the use of a variable step size that guarantees stability and convergence. This variable step size is analogous to the step size used in the ARMARKOV adaptive disturbance rejection algorithm discussed below. Unlike prior discrete-time controller as in [67], the convergence in [175] is based on a Lyapunov construction.

The problem of adaptive control of discrete-time systems requires special consideration. While the convergence proofs of [76, 144, 150] and many of the classical results for continuous-time systems are based on standard Lyapunov techniques, the corresponding theory for discrete-time systems is largely based on recursive least squares techniques (that is, Newton updates). The impediment to Lypunov-based adaptive control is discussed in [89]. Furthermore, the discrete-time theory is much less developed than the continuous-time theory; most of the known results are given in [67, 104, 117], while universal stabilizing discrete-time adaptive stabilization algorithms have been developed in [102, 103, 127].

In this project, we developed a Lyapunov method for full-state-feedback discrete-time stabilization in [175]. The basis for the stability proof is similar to the method used in [150], that is, the Lyapunov derivative is shown to be asymptotically nonpositive.

5 Adaptive Disturbance Rejection

Unlike adaptive stabilization and adaptive command following, adaptive cancellation is motivated by the desire to suppress disturbances. In practice, such disturbances can arise from a wide variety of sources. For example, rotating machinery can cause tonal or multi-tone disturbances, while turbulence can give rise to

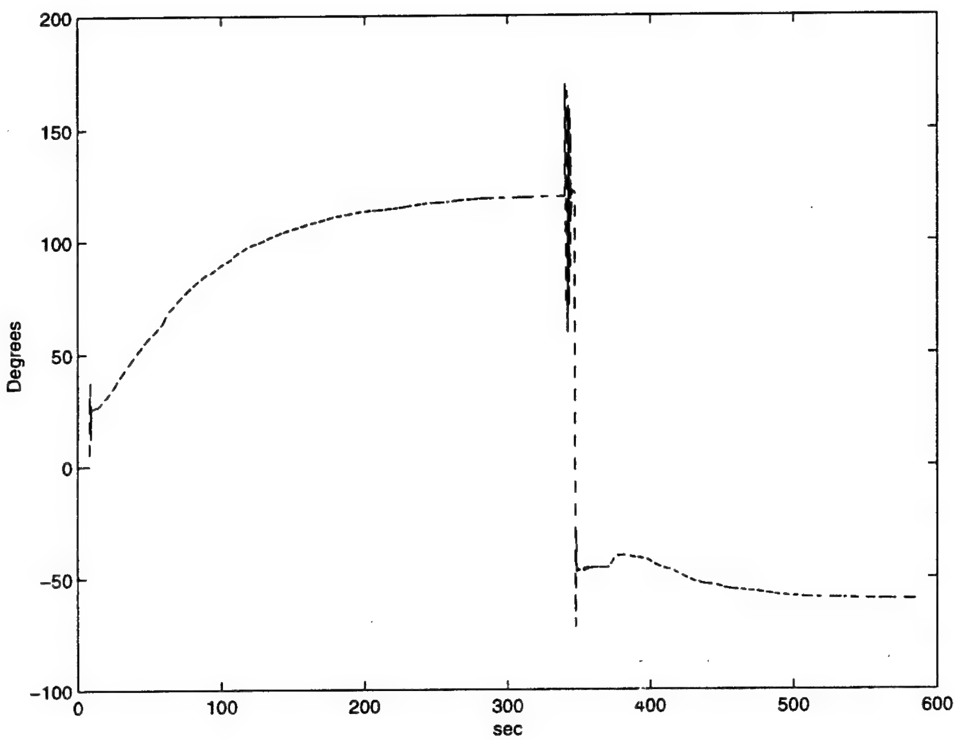

Figure 8: Response of the inner CMG gimbal with an abrupt change in angle command at time 350 sec. The Lyapunov-based adaptive stabilization control law estimated 21 mass, damping, and stiffness parameters to compensate for parametrically uncertain dynamics.

wide-band noise. The reduction of noise and vibration levels can be an important issue in aerospace vehicles. For example, aircraft engines can cause excessive noise and vibration, while helicopter blade motion can have a similar effect.

For tonal disturbances that are not measured, the classical approach to disturbance rejection is to use a controller that includes a model of the exogenous disturbance; this approach is based on the internal model principle. This technique has the significant property that neither the amplitude nor the phase of the disturbance need be known for asymptotic disturbance rejection. However, in order to implement such a controller, some plant modeling information is needed to assure stability. In fact, a root sensitivity argument involving shows that sign information is needed. An additional difficulty arises when the frequency of the disturbance is not known. A globally convergent algorithm for this problem (even in the presence of a fully modeled plant) remains open [32].

While fixed-gain controllers can be applied to vibration control problems, they are often cumbersome to apply in practice. In particular, fixed-gain methods require models of four transfer functions, namely, the transfer functions from control and disturbance inputs to performance and measurement outputs. In practice, it may be difficult to model or identify all of these transfer functions, especially when the disturbance enters the system in a spatially distributed manner and thus cannot be measured for identification purposes. In addition, changes in the plant dynamics and disturbance spectrum may necessitate extensive re-identification of the plant and redesign of the controller. Although robust control techniques can mitigate these difficulties, fixed-gain control techniques can be undesirable in the face of changing plant and disturbance conditions.

The adaptive disturbance rejection problem has not been widely studied in the classical adaptive control literature. However, it is widely studied in the active noise control literature under the guise of adaptive cancellation [61, 180]. This class of algorithms includes LMS (least mean square) algorithms with FIR and IIR controllers and numerous variants [12, 38, 53, 54, 58, 62, 86, 97, 105, 124, 141]. Since some of these algorithms assume special sensor and actuator arrangements and ignore the dynamics in the feedback path, they are often referred to as *feedforward algorithms*. Although many noise cancellation algorithms have been proposed (and patented), this literature lacks the rigor of mainstream control theory, and it is often difficult to ascertain the assumptions, the modeling and measurement requirements, and the performance guarantees afforded by the various algorithms.

With the ARMARKOV identification algorithm [6, 7] as our starting point, we developed the ARMARKOV adaptive cancellation algorithm in [172, 174]. The idea for basing controller synthesis on Markov parameters is not new, however. For example, fixed-gain Markov-parameter-based design is considered in [64, 65, 158]. The interesting aspect of Markov parameters is the fact that they are input-output quantities, that is, they are independent of the state space basis. This suggests that they can play a more fundamental approach in controller design than state space techniques.

The ARMARKOV adaptive disturbance rejection developed in [172, 174] is a direct adaptive control algorithm based upon ARMARKOV models. This control algorithm requires no prior knowledge of the disturbance spectrum and minimal modeling of the plant dynamics. Specifically, a model of the transfer function from control to performance variables is required, whereas no knowledge of the remaining transfer functions (from control to measurement, disturbance to performance, and disturbance to measurement) is needed. None of these transfer functions is assumed to be positive real, and the algorithm does not require accessibility of either the disturbance signal or the full state.

To perform laboratory experiments, we developed an acoustic testbed based on interconnected ducts (Figure 11) as well as an acoustic testbed based on a cylindrical drum (Figure 12). Using the duct testbed, we implemented ARMARKOV adaptive control for acoustic noise suppression with stationary and nonstationary disturbances, including single tones, multiple tones, white noise, and sine sweeps. In each case the controller had no prior knowledge of the nature of the disturbance, and only the (MIMO) transfer function from control inputs (speakers) to performance variables (microphones) was identified in advance. Figures 13 and 14 show typical experimental results for dual-tone and white noise disturbances after controller convergence.

An interesting feature of the ARMARKOV adaptive controller is the fact that it does not require a model of the control-to-measurement transfer function G_{yu}. (Note that some microphones are designated as perfor-

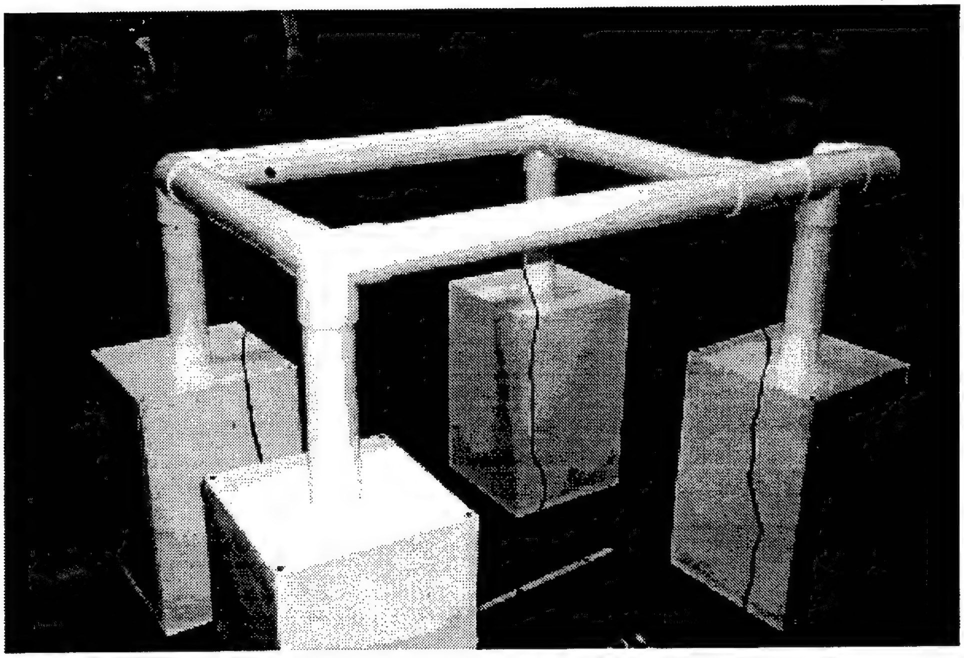

Figure 9: Acoustic duct experiment. This duct testbed was developed to test control laws for adaptive disturbance rejection, that is, disturbance rejection with unknown plant dynamics and disturbance spectrum.

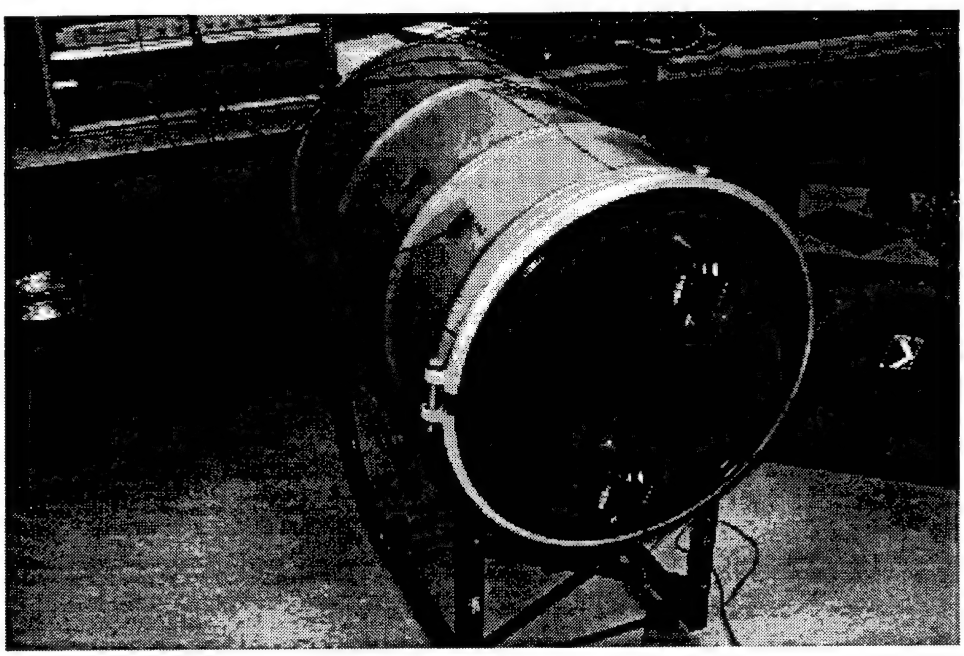

Figure 10: Acoustic drum experiment. This drum testbed was developed to test control laws for adaptive disturbance rejection and to develop multivariable linear identification methods.

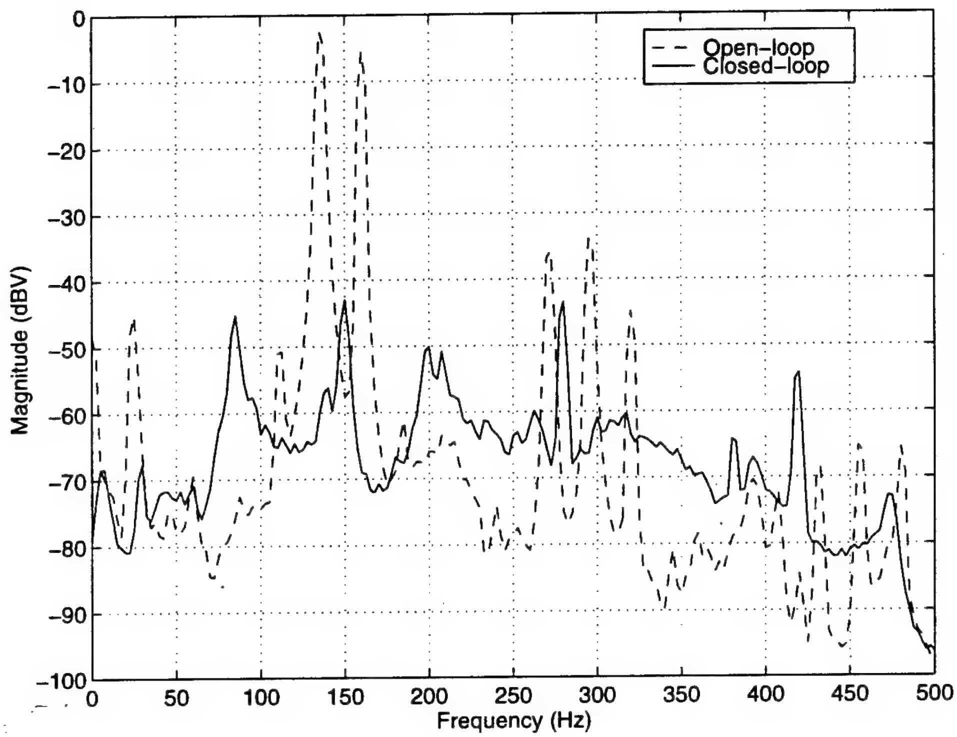

Figure 11: Adaptive disturbance rejection for a dual-tone disturbance. This plot shows the results of an active noise control experiment with dual-tone disturbance. The ARMARKOV adaptive disturbance rejection controller was able to suppress both tones without prior knowledge of the disturbance spectrum.

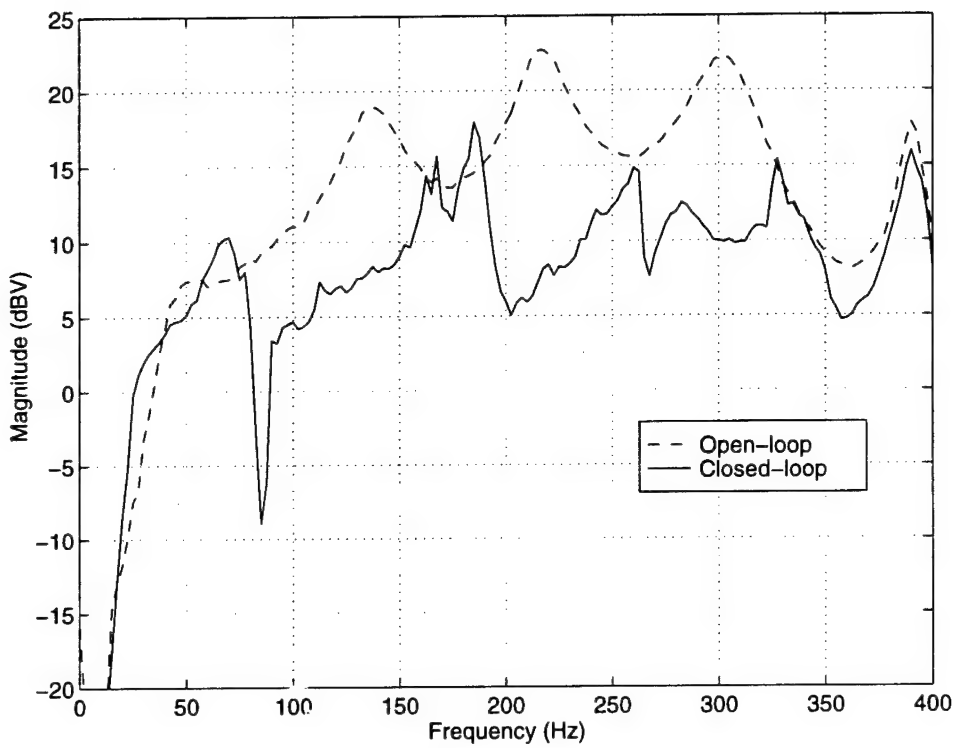

Figure 12: Adaptive disturbance rejection for a white noise disturbance. This plot shows the results of an active noise control experiment with white noise disturbance. As in Figure 4, the ARMARKOV adaptive disturbance rejection controller was able to suppress the disturbance without prior knowledge of the disturbance spectrum.

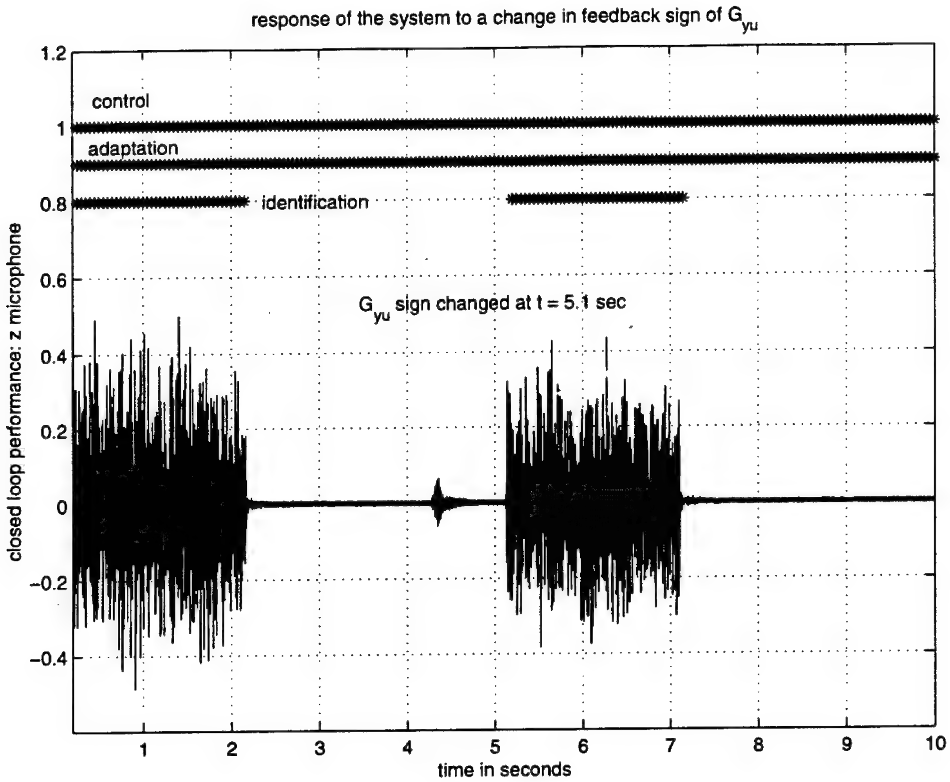

Figure 13: Simulated test of the ARMARKOV adaptive disturbance rejection algorithm. In this active noise control scenario, the sign of the feedback measurement signal was changed during operation causing an effective 180 degree phase shift in the loop transfer function. The response shows that the ARMARKOV adaptive disturbance rejection controller was able to re-stabilize the loop after the plant perturbation. Similar results have been observed experimentally.

mance variables for control-gain adaptation, while others are used as inputs to the (instantaneously linear) controller.) To investigate this feature, we examined via simulation and experiment the ability of the algorithm to adapt to changes in G_{yu} during operation [151, 152]. In one scenario reported in [151], a sign change was inserted in the feedback loop to effect 180 degrees of phase shift in the loop transfer function at every frequency. As expected, with the adaptation disabled after initial convergence (that is, with the control gains frozen after initial operation) the control system was destabilized. However, with the adaptation remaining operational the algorithm recovered from this perturbation and continued to reject both single-tone and dual-tone disturbances (see Figure 15). This aspect has been demonstrated both numerically and experimentally. In another scenario, the connections to sensors y_1 and y_2 were suddenly reversed during operation. Again, the algorithm demonstrated the ability to recover from this perturbation and adapt to the modified plant.

To further decrease the need for off-line modeling, we extended ARMARKOV adaptive disturbance rejection to include concurrent identification of the control-to-performance transfer function. This hybrid algorithm is model free in the sense that no prior, off-line modeling or identification is required for implementation. In fact, in all numerical and experimental studies, the controller and control-to-performance transfer functions were initialized to zero. A supervisory controller developed in [151, 153] was used to perform mode switching between control on/off, adaptation on/off, identification on/off, and control gain resetting.

The identification and control algorithms as well as all functions of the supervisory controller were implemented, tested, and demonstrated using a multiprocessor dSPACE system. The key issue in the design of the supervisory controller is the decision logic for distinguishing between a change in the disturbance spectrum and a change in the control-to-performance transfer function. In the latter case, the supervisory

controller employs an identification signal to update the model of this transfer function. The ability to perform this identification depends on the identification signal level relative to the ambient plant disturbance. Experimental results are described in [151, 153].

We also applied ARMARKOV adaptive control to noise control using a pneumatic flow actuator [147]. This actuator provides an experimental test of the ability of the algorithm to operate in the presence of unmodeled nonlinear actuator dynamics. The same actuator was also used for active control of the acoustic response of a ducted flame [101]. The experimental setup involves a burner in a 4-foot tube with propane fuel and servovalve actuation of the air flow through the burner. The fundamental acoustic mode was suppressed 20 dB without analytical modeling.

In related work we considered the relationship between fixed-gain control and adaptive feedforward control implemented concurrently. Specifically, in [44] we showed that the optimality of LQG control holds independently of the algorithm used for adaptive feedforward control. Finally, our studies of active noise control were supported by acoustic analysis of boundary control in 1-D and 2-D ducts [148, 149, 173].

Our acoustic experiments have helped us to understand the relationship between feedforward and feedback strategies in active noise control in [74, 116, 156]. In particular, the analysis in [74, 116] suggests that the post-adaptation performance of feedforward algorithms can largely be attributed to the spatial location of the sensor, actuator, disturbance, and performance signals, which precludes nonminimum phase zeros in the disturbance-to-measurement and control-to-performance paths. This arrangement, which is responsible for the "feedforward" interpretation held by the acoustic control community, permits asymptotically perfect cancellation as predicted by singular LQG theory [98]. The analysis in [74] also sheds light on the spillover phenomenon in terms of Bode integral constraints.

As already discussed, the adaptive cancellation problem is distinct from the adaptive stabilization and command following problem in several respects. For adaptive cancellation, the plant is usually assumed to be open-loop stable, so that adaptive stabilization is not required. However, the exogenous disturbance is usually assumed to be unmeasured with unknown spectrum, and this introduces an additional level of uncertainty into the problem. In this case, some degree of plant modeling information is assumed, usually the secondary path transfer function.

6 Nonlinear Control

In the area of nonlinear control we focused on two concepts for continuous-time systems, namely, finite-time stability and semistability. Our research on these concepts involves fundamental extensions to existing theory with practical ramifications.

Finite-time stability refers to system behavior in which the state converges to an equilibrium in finite time. Such controllers involve fractional-order dynamics, which apply high authority in the neighborhood of an equilibrium to achieve faster-than-exponential convergence. Note that this local high authority does not impact saturation constraints.

The classical minimum-time controller for the double integrator yields finite-time convergence. However, this controller is discontinuous, whereas our interest is in continuous feedback controllers. Furthermore, finite-time stabilization using time-varying controllers is studied in [45]. However, we consider only time-invariant controllers.

Since finite-time convergence is faster than exponential convergence, it can be seen that finite-time convergent dynamics are necessarily sublinear (fractional powers) and thus they are non-Lipschitzian at the equilibrium. Normally, the lack of Lipschitz continuity of the vector field would yield nonuniqueness of solutions. However, with finite-time stability, an equilibrium is an attractor, which rules out nonuniqueness. In a related investigation, we studied the consequences of non-Lipschitzian dynamics for nonuniqueness within the context of classical dynamics [21].

In [23] we derived finite-time-stabilizing controllers for the rotational and translational double integrators, where, in the latter case, we accounted for the fact that the state space is a compact manifold (see also [25]). These controllers were derived by phase plane constructions that drove the state to a continuous sliding manifold in finite time. Next, we developed a general theory of finite-time stability in [20, 26], where we studied the regularity of the settling-time function and developed Lyapunov and converse Lyapunov theorems.

The difficulty of constructing Lyapunov functions for finite-time stability suggested an alternative path, which we pursued in [22, 27]. Specifically, in [22, 27] we confined our attention to homogeneous systems and obtained the result that an asymptotically stable homogeneous system is finite-time stable if and only if it has negative degree. Using these results we obtained an explicit construction for finite-time-stabilizing controllers for scalar input linear systems.

Semistability refers to the convergence of trajectories to a Lyapunov stable equilibrium that may depend on the initial condition and which need not be asymptotically stable (see Figure 16). This notion of stability applies to systems with a continuum of equilibria such as a dissipative mechanical system including systems with friction [8, 19]. Additional examples of semistable systems include aircraft lateral dynamics (traditionally called weathercock stability) and systems involving mass and energy balance (known generally as compartmental models [17, 85]). For linear systems, semistability refers to systems with poles in the open left half plane or at the origin, where the pole at the origin (if present) is semisimple (nondefective). In transfer function form, this class of systems is precisely the class of systems to which the classical final value theorem is valid.

In earlier research we analyzed matrix second-order models for mechanical systems that are semistable. The rigid body modes of such systems are necessarily damped, and tests for semistability were given in [15]. More recently, we developed Lyapunov theory for nonlinear semistable systems in [24]. This theory is based on a general definition of semistability in which the system is convergent and each equilibrum is assumed to be Lyapunov stable. Lyapunov tests for semistability were developed, and these conditions show that semistability lies between Lyapunov stability and asymptotic stability.

As an application of semistability, we considered the stability of mass action kinetics in [16]. These equations model the dynamics of chemical reactions, which are a special class of compartmental models. Since such systems model chemical concentrations, the state is nonnegative and the stability theory is applied to the nonnegative orthant as an invariant set. It was shown in [16] that the notion of semistability applies to the zero deficiency theorem of Feinberg [57]. This theorem provides rate-independent structural conditions under which the system is guaranteed to be semistable.

As another application of the concept of semistability, we constructed semistable systems that exhibit hysteresis. This hysteretic behavior arises from the fact that, since the system possesses a continuum of equilibria, the state of the system effectively converges to a sequence of equilibria if its dynamics are sufficiently fast relative to the frequency content of the input. Although this model behavior is not new (see [91, 66]), the concept of semistability has provided a framework for systematically examining and exploiting this model for hysteresis. Figure 17 illustrates the hysteretic response of a semistable system with low frequency input.

From a practical point of view, semistable models for hysteresis have several benefits. Specifically, these models are finite-dimensional and thus are conceptually simpler than operator models such as the Preisach model [115]. Furthermore, for higher frequency inputs the response will have a different character, and model parameters can be set to match the desired system behavior. Finally, this framework provides an appropriate interpretation of hysteresis as a system-theoretic phenomenon, namely, as the quasi-steady-state input-output behavior of a semistable system which naturally exhibits memory-dependent behavior.

We also considered nonlinear control of mechanical systems involving shape-change actuation, that is, dynamic mass distribution, to effect attitude control through geometric phase. Our research in this area is experimental in conjunction with A. Bloch and N. H. McClamroch [31, 142]. To this end, we developed two testbeds for spacecraft attitude control. One testbed, which is based on an air spindle, allows single-axis motion (Figure 17), while the other testbed is based on a triaxial air bearing (Figure 18). These testbeds will allow us to implement and demonstrate controllers that recognize conservation of angular momentum.

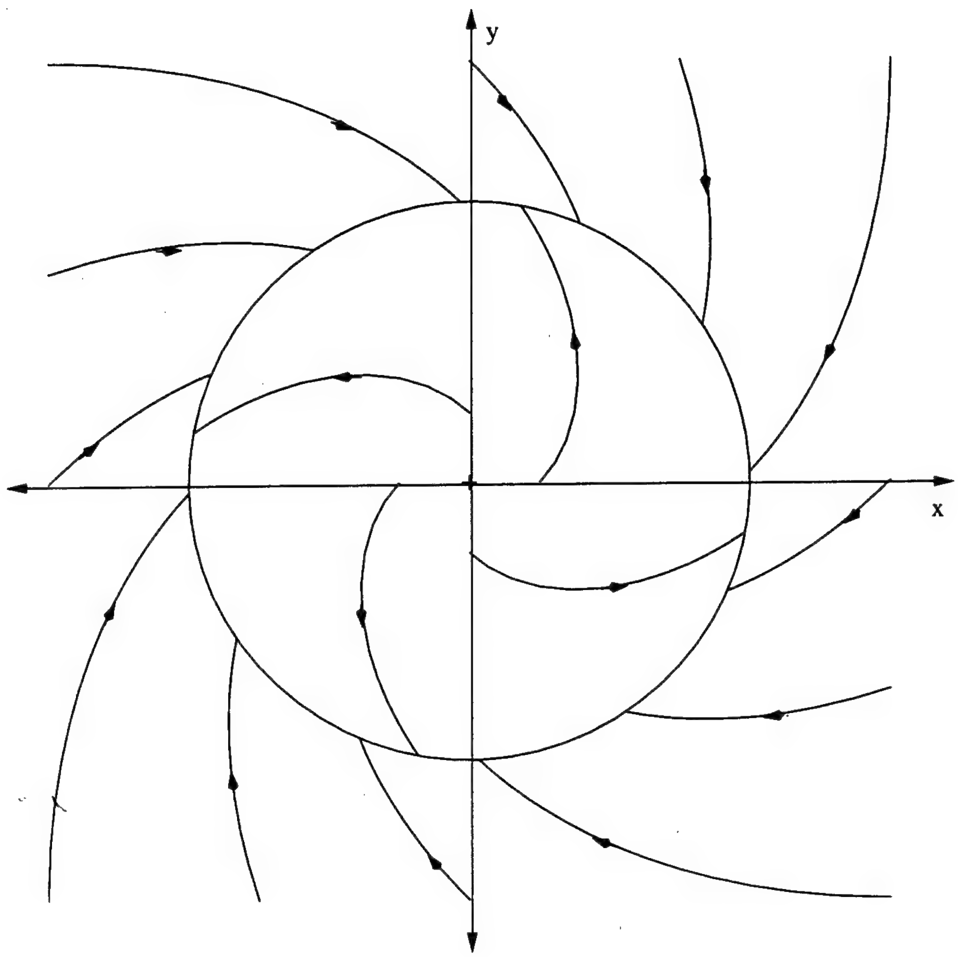

Figure 14: Phase portrait demonstrating convergence of all trajectories to the unit circle, where each point is a Lyapunov stable equilibrium. A Lyapunov test guarantees that the system is semistable.

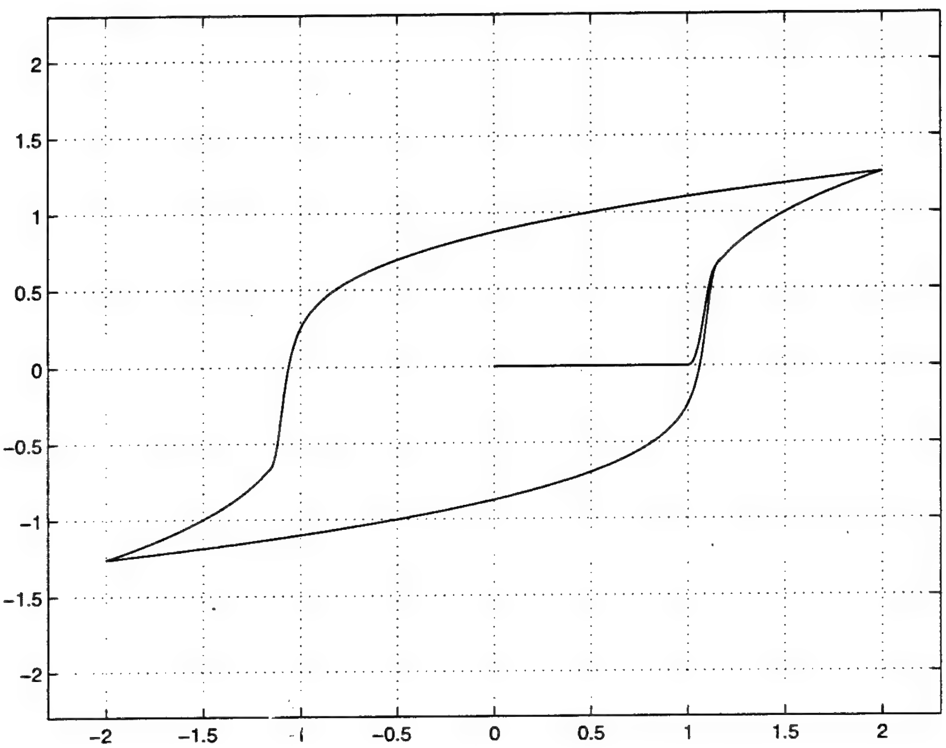

Figure 15: Example of hysteretic behavior based on semistability theory. This plot shows the hysteretic response of a finite-dimensional nonlinear semistable system. The figure was produced by plotting the output versus the input of the system for a low-frequency input. The hysteresis characteristic is independent of the detailed time history of the input (for example, it may be sinusoidal or triangular).

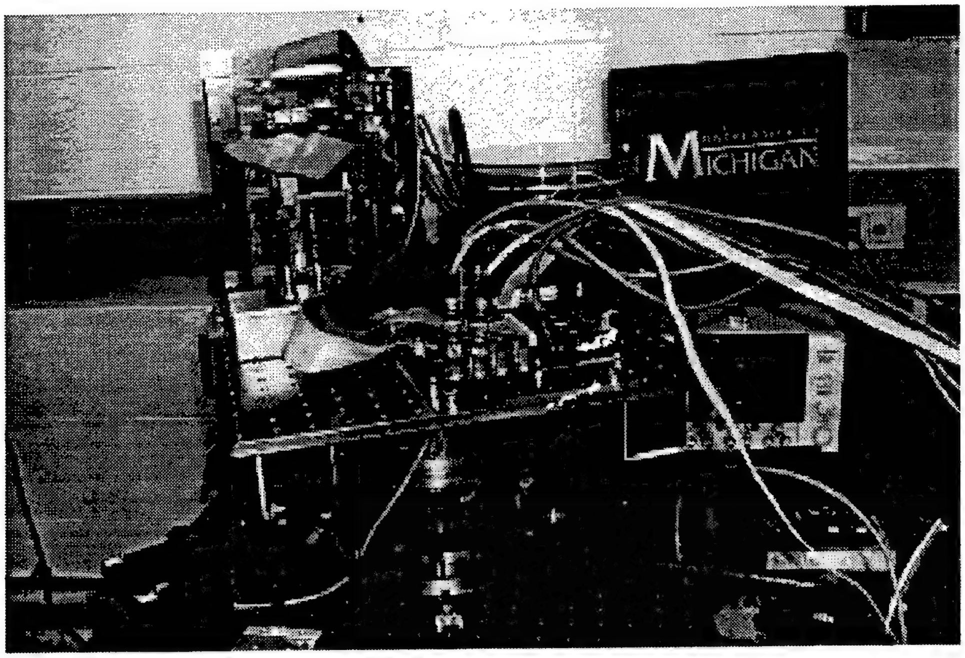

Figure 16: 1D spacecraft experiment. This testbed, which is based on a low-friction air spindle, was developed to emulate one-dimensional rotational dynamics. Here the testbed is configured with a reaction wheel actuator. The research objective is to effect attitude control using shape change actuation, which is realized with linear actuators. Operation is untethered with a wireless ethernet link.

Reaction wheels and linear translational actuators will be used ([18]).

7 Technology Transfer

A patent application was filed for a noise and vibration system based on the ARMARKOV adaptive disturbance algorithm developed in [174]. We received notification in 11/00 that this application was approved.

The principal investigator visited industrial and Governmental organizations to discuss technical issues relating to the AFOSR grant. AFOSR-supported research was presented in seminars given at the following organizations: Aerospace Corporation (twice), Air Force Research Laboratories in Albuquerque, Hughes (3 times), TRW (twice), NASA Langley (twice), Wright-Patterson Air Force Base, Rockwell Science Center, Lord Corporation, HR Textron, Ford (3 times), JPL (twice), Honeywell Tech Center in Minneapolis, and Honeywell Commercial Aircraft Division in Phoenix.

The most significant instances of technology transfer involved our interactions with Lord Corporation, NASA Langley, Hughes, HR Textron, and Honeywell. In a project supported by Lord Corporation, we tested ARMARKOV adaptive disturbance rejection controllers on an aircraft fuselage located in their facility in Cary, NC. Using our dSPACE real-time processor, we demonstrated the feasibility of rejecting multiple-tone disturbances within a 3-dimensional acoustic space with minimal prior modeling.

For a Phase I SBIR sponsored by NASA Langley we served as a subcontractor under PSI, Inc. The goal of this project was to study active noise control algorithms to suppress fan noise on an engine testbed. To do this, we implemented adaptive disturbance rejection controllers on a testbed located at NASA Langley. In these experiments we obtained good suppression of a large number of disturbance harmonics over a broad frequency band.

As a result of several visits to Hughes Space and Communications in El Segundo, we developed an adaptive

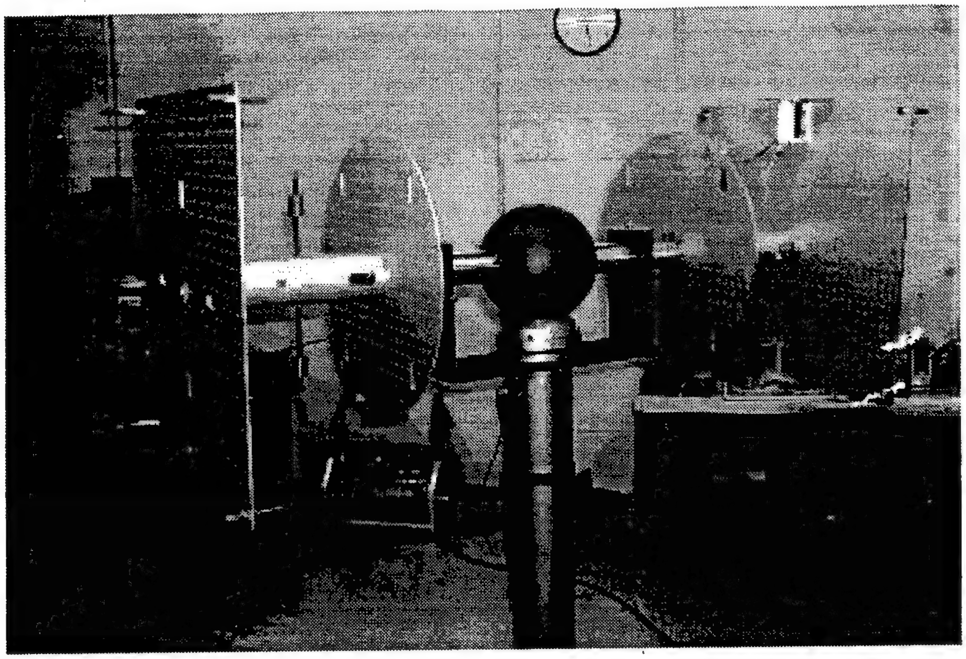

Figure 17: 3D spacecraft experiment. This testbed, which is based on a low-friction triaxial air bearing, was developed to emulate three-dimensional rotational dynamics. Unrestricted motion is allowed in roll and yaw, with +/- 45 degree motion allowed in pitch. Actuation is provided by reaction wheels and shape change actuators.

spacecraft controller for active nutation control [3]. This problem was motivated by the need to stabilize rotating spacecraft during the deployment of appendages when mass distribution is changing and uncertain. These visits also stressed the importance of reliable controllers for stabilizing the double integrator plant under various constraints. Our detailed study in [139, 140] was motivated by this interaction. The study in [139, 140] provides an extensive comparison of a diverse collection of controllers developed under AFOSR support (including the results of [22, 23, 27, 35, 36]) as well as controllers from the nonlinear and adaptive literature [81, 164].

In a project funded by HR Textron, we developed controllers for servopneumatic applications, which use compressed air for precision motion control. This project allowed us to develop a dSPACE/Simulink-based motion control toolbox that allows the user to test and tune the controllers that are compared in [139, 140].

For Honeywell Tech Center and Honeywell Commercial aviation, we analyzed the actuator/cable dynamics of control components for business-class jets. To do this, we designed and constructed an experimental testbed which allowed us to perform identification and control experiments. Using nonlinear identification techniques we obtained a detailed model of the back emf, feedback dissipation, and hysteretic characteristics of the aircraft actuator. This work motivated the semistable hysteresis modeling approach of [100] as well as the describing function analysis of the anti-backlash controller given in [165].

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